HEAT TRANSFER IN Z-SHAPED HEAT EXCHANGERS

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The characteristics of heat transfer in Z-shaped heat exchangers are considered. The parameters determining the thermal efficiency of such heat exchangers are described, and generalized relationships for practical calculations are derived. The basic results have been confirmed by an experimental investigation.

It is virtually impossible to realize in pure form the most efficient coolant counterflow system in regenerative heat exchangers: sections with crossflow inevitably appear in places where the working medium is supplied as a result of structural requirements (the so-called Z-shaped heat exchanger).

In heat exchangers of stationary gas-turbine plants with long channels (of the order of several meters), these sections have a relatively small extent, so that their effect on the heat transfer process is usually neglected.

In compact heat exchangers, which are used fairly widely at present, the use of channels with small equivalent diameters reduces the extent of these sections (their order of magnitude is 0.1-0.2 m). The crossflow sections in these devices can add up to a considerable percentage of the total heat-exchange surface (up to 40-60%), so that they affect considerably the amount of transferred heat.

The results of an investigation of heat transfer in Z-shaped heat exchangers are given below.

The scheme of a Z-shaped heat exchanger is shown in Figs. 1 and 3. In part II, air and gas move by counterflow, while, in parts I and III, they move by crossflow.

The basic assumptions are the following:

1) the heat exchange coefficients in all parts are assumed to be constant;

2) parts I and III have a triangular shape in the plan view, and their geometric and thermal parameters are equal ($F' = F'''$, $k' = k'''$).

3) the distribution of discharge over the transverse cross section is assumed to be uniform in all parts;

4) the coolant mixing conditions are considered for two extreme cases: a) there is no mixing of coolants in the cross sections normal to the flow in any of the parts; b) in parts I and III the air temperature is completely equalized in every cross section as a result of mixing; there is no mixing of air in part II and no mixing of gas throughout the heat exchanger.

System without Air Mixing. The heat transfer in a Z-shaped heat exchanger (Fig. 1) is described by a system of six equations (these relationships have been derived on the basis of the heat transfer and heat balance equations, written for elementary areas of parts I and III of the heat exchanger; the variation in the temperatures of the working media in part II is determined by means of the well-known relationships for counterflow):

$$T_g' - T_a' = -\frac{1}{2M_g'} \frac{\partial T_g'}{\partial y'}, \tag{1}$$

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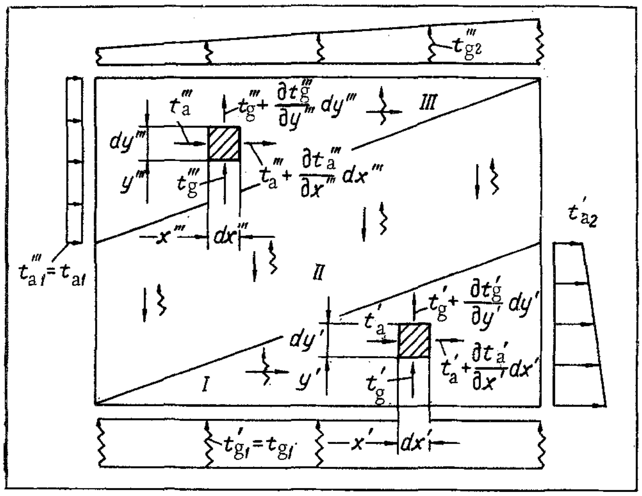

Fig. 1. Calculation scheme for a Z-shaped heat exchanger without air mixing.

$$T_g' - T_a' = \frac{1}{2\omega M_g'} \frac{\partial T_a'}{\partial x'}, \tag{2}$$

$$T_g''' - T_a''' = -\frac{1}{2M_g'} \frac{\partial T_g'''}{\partial y'''}, \tag{3}$$

$$T_g''' - T_a''' = \frac{1}{2\omega M_g'} \frac{\partial T_a'''}{\partial x'''}, \tag{4}$$

$$T_{g1}'' - T_{g2}'' = \eta_g'' \left(T_{g1}'' - T_{a1}''\right), \tag{5}$$

$$T_{a2}'' - T_{a1}'' = \omega \left(T_{g1}'' - T_{g2}''\right). \tag{6}$$

According to [1], the efficiency of the counterflow part II is

$$\eta_g'' = \begin{cases} M_g''/(1+M_g'') & \text{for } \omega = 1, \\ \dfrac{1-\exp[(\omega-1)M_g'']}{1-\omega\exp[(\omega-1)M_g'']} & \text{for } \omega \neq 1. \end{cases} \tag{7}$$

Using a computer to solve the system of Eqs. (1)-(6) according to the method of finite differences, we have found the gas and air temperature profiles T_{g2}''' and T_{a2}', respectively, and determined the mean temperature of the working media $T_{g2} = \int_0^1 T_{g2}''' dx'''$ and $T_{a2} = \int_0^1 T_{a2}' dy'$ at the heat exchanger outlet. The degree of regeneration for a Z-shaped heat exchanger was determined with respect to the temperature change of the working medium with the smaller water equivalent:

$$\eta_r = \begin{cases} 1 - T_{g2} & \text{for } \omega \leqslant 1, \\ T_{a2} & \text{for } \omega \geqslant 1. \end{cases} \tag{8}$$

Analysis of Eqs. (1)-(6) shows that the degree of regeneration depends on three quantities: ω, M_g', and M_g''. For convenience in comparing the efficiency of the Z-shaped system with the efficiency of other systems, the equivalent quantities M and $\overline{M}$, calculated with respect to the parameters of the coolant with the smaller water equivalent, were used instead of the parameters M_g' and M_g'' in the final results:

$$M = \begin{cases} 2M_g' + M_g'' & \text{for } \omega \leqslant 1, \\ \omega\left(2M_g' + M_g''\right) & \text{for } \omega \geqslant 1. \end{cases} \tag{9}$$

$$\overline{M} = 2M_g'/(2M_g' + M_g''). \tag{10}$$

TABLE 1. Degree of Regeneration of a Z-Shaped Heat Exchanger without Air Mixing

M \ $\overline{M}$	ω=1						ω=0.5 and 2				ω=0.25 and 4		
	0	0.1	0.3	0.6	0.8	1.0	0	0.3	0.6	1.0	0	0.6	1.0
1	0,500	0,500	0,497	0,489	0,483	0,476	0,565	0,562	0,557	0,547	0,598	0,594	0,588
2	0,667	0,666	0,660	0,643	0,629	0,614	0,775	0,769	0,756	0,732	0,823	0,812	0,797
3	0,750	0,749	0,741	0,719	0,701	0,681	0,874	0,868	0,851	0,820	0,919	0,906	0,888
4	0,800	0,798	0,789	0,766	0,745	0,722	0,927	0,921	0,904	0,870	0,965	0,951	0,934
6	0,857	0,855	0,846	0,821	0,799	0,772	0,974	0,969	0,956	0,924	0,992	0,986	0,974
8	0,889	0,887	0,878	0,854	0,831	0,802	0,991	0,988	0,978	0,951	0,998	0,996	0,989
10	0,909	0,907	0,898	0,875	0,852	0,822	0,998	0,995	0,989	0,967	1,0	0,999	0,995
15	0,938	0,935	0,928	0,908	0,886	0,855	1,0	0,999	0,998	0,986	1,0	1,0	0,999

TABLE 2. Degree of Regeneration of a Z-Shaped Heat Exchanger with Air Mixing

M \ $\overline{M}$	ω=0.25				ω=0.5					ω=1					
	0	0.6	0.8	1.0	0	0.3	0.6	0.8	1.0	0	0.1	0.3	0.6	0.8	1.0
1	0,598	0,593	0,589	0,586	0,565	0,562	0,556	0,550	0,543	0,500	0,500	0,497	0,487	0,479	0,471
2	0,823	0,808	0,797	0,784	0,775	0,768	0,750	0,732	0,711	0,667	0,666	0,658	0,636	0,614	0,590
3	0,919	0,900	0,882	0,860	0,874	0,866	0,840	0,812	0,777	0,750	0,749	0,739	0,707	0,675	0,635
4	0,965	0,943	0,923	0,893	0,927	0,919	0,889	0,855	0,807	0,800	0,798	0,787	0,750	0,710	0,658
6	0,992	0,978	0,958	0,919	0,974	0,967	0,939	0,900	0,835	0,857	0,855	0,842	0,800	0,752	0,678
8	0,998	0,990	0,974	0,930	0,991	0,986	0,964	0,925	0,847	0,889	0,887	0,874	0,831	0,778	0,687
10	1,0	0,995	0,982	0,936	0,998	0,994	0,977	0,942	0,854	0,909	0,907	0,894	0,853	0,798	0,692
15	1,0	0,999	0,993	0,944	1,0	0,999	0,992	0,967	0,863	0,938	0,935	0,924	0,887	0,833	0,698

M \ $\overline{M}$	ω=2					ω=4			
	0	0.3	0.6	0.8	1.0	0	0.6	0.8	1.0
1	0,565	0,562	0,556	0,551	0,545	0,598	0,593	0,590	0,587
2	0,775	0,769	0,753	0,738	0,722	0,823	0,811	0,803	0,794
3	0,874	0,867	0,845	0,824	0,798	0,919	0,905	0,894	0,882
4	0,927	0,920	0,896	0,871	0,838	0,965	0,949	0,939	0,924
6	0,974	0,968	0,947	0,920	0,876	0,992	0,984	0,976	0,962
8	0,991	0,987	0,971	0,944	0,894	0,998	0,994	0,989	0,976
10	0,998	0,994	0,983	0,959	0,903	1,0	0,998	0,994	0,983
15	1,0	0,999	0,995	0,978	0,913	1,0	1,0	0,999	0,990

TABLE 3. Test Results for Z-Shaped Heat Exchangers

№ 1 ($\overline{M}\approx 1$)			№ 2 ($\overline{M}\approx 0.4$)			№ 3 ($\overline{M}\approx 0.6$)			№ 4 ($\overline{M}\approx 0.3$)		
G, kg/sec	η_r exp	η_r calc	G, kg/sec	η_r exp	η_r calc	G, kg/sec	η_r exp	η_r calc	G, kg/sec	η_r exp	η_r calc
0,00835	0,605	0,610	0,00827	0,662	0,659	0,00820	0,655	0,651	0,00825	0,621	0,633
0,00692	0,627	0,632	0,00692	0,687	0,685	0,00681	0,679	0,674	0,00690	0,647	0,657
0,00566	0,656	0,657	0,00545	0,717	0,714	0,00556	0,701	0,699	0,00556	0,667	0,684
0,00420	0,681	0,684	0,00412	0,743	0,748	0,00408	0,732	0,732	0,00422	0,696	0,719

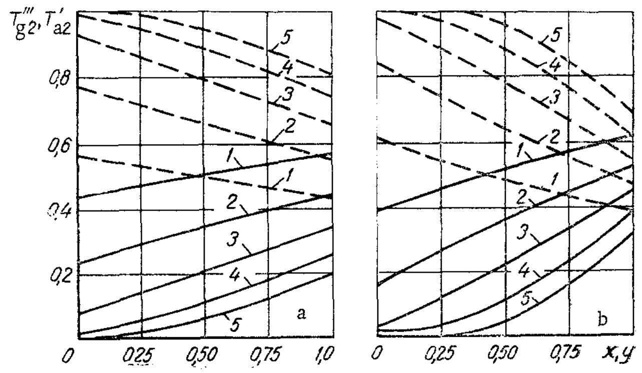

Fig. 2. Gas and air temperature profiles at the outlet of a Z-shaped heat exchanger without air mixing ($\omega = 1$; the solid curves represent $T'''_{g2} = (t'''_{a2} - t_{a1})/(t_{g2} - t_{a1})$; the dashed curves represent $T'_{a2} = (t'_{a2} - t_{a1})/(t_{g1} - t_{a1})$): 1) M = 1; 2) 2; 3) 4; 4) 8; 5) 15; a) $\bar{M} = 0.3$; b) 0.6.

The calculation results are given in Table 1; Fig. 2 shows the most characteristic temperature profiles at the outlet of a Z-shaped heat exchanger.

For $\bar{M} = 0$, a Z-shaped heat exchanger constitutes a pure counterflow heat exchanger, whose efficiency is described by relationships (7). For $\bar{M} = 1$, the internal boundaries of parts I and III coincide (Fig. 1), which results in a single-pass crossflow heat exchanger without coolant mixing. In this case, the results are in satisfactory agreement with the solution for this system given in [2] (the discrepancy between the values of η_r is less than 0.001).

With respect to thermal efficiency, a Z-shaped system occupies the intermediate position between pure counterflow ($\bar{M} = 0$) and single-pass crossflow ($\bar{M} = 1$).

The Z-shaped system has the best characteristics for small values of the $\bar{M}$ parameter: $\bar{M} = 0.1$-0.3. However, in compact heat exchangers, the realization of a coolant motion approaching counterflow conditions involves considerable design difficulties and is therefore not always possible.

As the zones with coolant crossflow expand, the value of $\bar{M}$ increases, and the system's efficiency decreases. Nevertheless, even for large $\bar{M}$ values, $\bar{M} = 0.5$-0.7, the thermal efficiency of a Z-shaped system is not inferior to that of multipass crossflow systems (with two to three sections), which makes it one of the most promising systems for heat exchangers with a high degree of regeneration ($\eta_r > 0.70$-0.75).

System with Air Mixing. In contrast to the system just considered, the air temperatures in parts I and III are a function of the argument x only (Fig. 3).

The system of equations describing the heat transfer process consists of six relationships:

$$x\,dT'_a + (T'_a - T''_{a2})\,dx + dT'_a\,dx = \omega(1 - T'_a)[1 - \exp(-2M'_g x)]\,dx, \tag{11}$$

$$1 - T'_{g2} = (1 - T'_a)[1 - \exp(-2M'_g x)], \tag{12}$$

$$(1 - x)\,dT'''_a = \omega(T'''_{g1} - T'''_a)\{1 - \exp[-2M'_g(1 - x)]\}\,dx, \tag{13}$$

$$T'''_{g1} - T'''_{g2} = (T'''_{g1} - T'''_a)\{1 - \exp[-2M'_g(1 - x)]\}. \tag{14}$$

Expressions (5) and (6) are used as the fifth and sixth equations.

Relationships (11) and (13) are heat balance equations, written for elementary areas dx of parts I and III (Fig. 3). In these expressions, the amount of heat supplied to air dQ is determined by means of Eqs. (12) and (14), which describe the gas temperature variation within parts I and III for $t'_a = \text{const}$ and $t'''_a = \text{const}$ at the section x under consideration [1].

The system of equations (11)-(14), (5), and (6) was solved by means of a computer, using the method of finite differences. The methods of processing and presentation of the results used for the system considered previously were also used in this case.

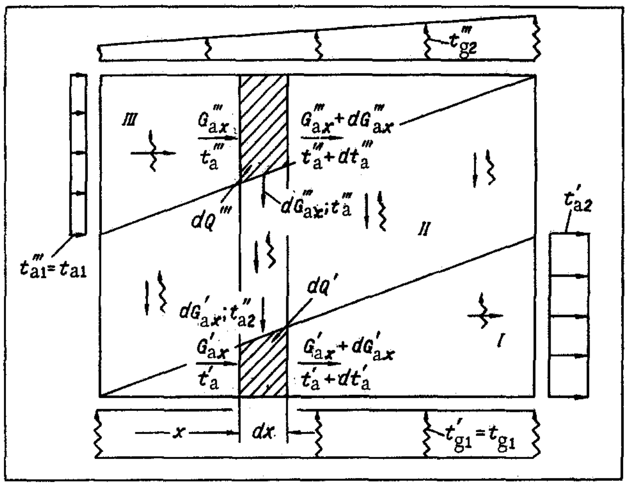

Fig. 3. Calculation scheme for a Z-shaped heat exchanger with air mixing.

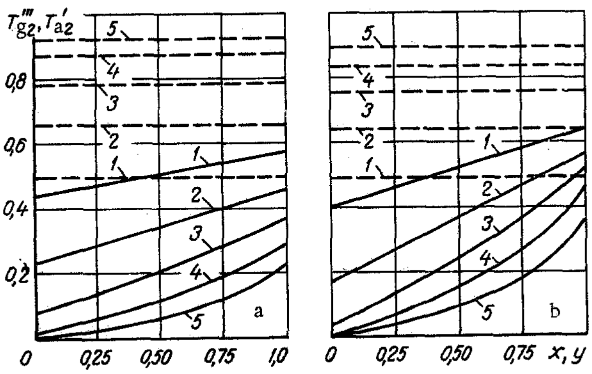

Fig. 4. Gas and air temperature profiles at the outlet of a Z-shaped heat exchanger with air mixing. The notation is the same as in Fig. 2.

The calculation results are given in Table 2 and Fig. 4.

For $\bar{M} = 1$, a Z-shaped heat exchanger (Fig. 3) is transformed into a single-pass crossflow heat exchanger with mixing of one of the coolants. The air mixing conditions constitute a specific feature of such a heat exchanger. The mixing in part I is independent of the mixing in part III, and the air temperatures in these parts are different at the section x = const. For this reason, the η_r value for the particular case considered above ($\bar{M} = 1$) somewhat exceeds the η_r values found in [3] for a crossflow system with complete mixing and, consequently, equal air temperatures at any transverse cross section x = const.

Analysis of the data given in Tables 1 and 2 indicates that air mixing in the crossflow zones reduces the efficiency of a Z-shaped system. This effect is most strongly pronounced for $\bar{M} > 0.3$-0.4.

Four heat exchangers made of soldered flat and corrugated plates were prepared for the tests. All the experimental specimens had identical channels in the gas cavity: straight channels 119 mm long with an approximately triangular shape and smooth walls. A similar heat-exchange surface was also used in the air cavity of the heat exchanger. The difference consisted only in the arrangement of the corrugated plates, which made it possible to realize different schemes of coolant motion.

Heat exchanger No. 1 had straight channels with a length of 199 mm in its air cavity; it corresponded to the single-pass crossflow system. Heat exchangers Nos. 2 and 3 were based on the Z-shaped system with double deflection of the air flow through 90° (Fig. 1); the channel lengths in the counterflow part of the matrix were equal to 82 and 49 mm, respectively. The only difference between heat exchangers Nos. 4 and 2 was the absence of finning on parts with counterflow in the air cavity (slot channels).

The tests were performed in a device where the air (293°K) flowed first through the air cavity and then through the gas cavity of the heat exchanger after heating in an electric heater (to 400°K). The equality of the water equivalents of the coolants ($\omega = 1$) was thus maintained automatically.

The air discharge was measured by means of a calibrated diaphragm with an accuracy to ±1%, the mean-mass air and "gas" temperatures ahead and beyond the heat exchanger were measured by means of nickel resistance thermometers in conjunction with a UMV bridge, which ensured a measuring accuracy of 0.2°C, while the temperature profiles were measured by means of Chromel–Copel thermocouples (thermoelectrode diameter, 0.1 mm) and recorded with a PP-2 potentiometer. The heat loss to the external medium was measured by means of the additional-wall method, for which foam-plastic plates with a thickness of 8 mm and copper resistance thermometers pasted on their surfaces were used; the plates were placed on both sides of the experimental specimen in a layer of thermal insulation (felt).

The test results are given in Table 3. The column $\eta_{r\,exp}$ provides the experimental values of the degree of regeneration, equal to the mean-arithmetic values of η_r, which are calculated separately with respect to changes in the temperature of the working medium in the air and gas cavities of the heat exchanger. A correction for the heat loss to the external medium was used in calculating η_r (for all heat exchangers in our experiments, this loss was less than 0.7% of the total amount of transferred heat).

The column $\eta_{r\,calc}$ of Table 3 provides the calculated values of the degree of regeneration, determined by means of the relationships for the system without air mixing (Table 1). The heat exchange coefficients in triangular channels were determined with respect to the characteristics of a similar surface on a special stand, while the heat exchange coefficients in the slot channels of heat exchanger No. 4 were determined with respect to data from [2].

Tests of heat exchanger No. 1 ($\bar{M} = 1$) have confirmed the adequacy of the proposed experimental method and the satisfactory accuracy of the data obtained. The heat balance with respect to air and "gas" was accurate within 2-3%.

In testing the Z-shaped heat exchangers, Nos. 2 and 3, the experimental and theoretical data on the degree of regeneration and the air and "gas" temperature distributions at the outlet from the matrix showed satisfactory agreement (Table 3). The continuous longitudinal finning virtually precluded the possibility of transverse mixing of coolants. Therefore, the relationships for a system without air mixing can be used in calculating such heat exchangers (Table 1 and Fig. 2).

The absence of finning in parts with crossflow in heat exchanger No. 4 reduced considerably (by a factor or approximately 2.5-3.0) the hydraulic loss in Z-shaped channels, but it produced nonuniformities in the air distribution in channels of the matrix. This was indicated by direct measurements of the velocity field at the outlet from the Z-shaped channels and also by the discrepancy between the measured and the calculated "gas" temperature profiles at the heat exchanger outlet (for comparison, one can point out the satisfactory agreement between these profiles in tests of heat exchanger No. 2). The nonuniformity of the air distribution reduced the degree of regeneration of heat exchanger No. 4 in comparison with the calculated value (Table 3).

Moreover, tests of heat exchanger No. 4 have revealed a considerable nonuniformity of the air temperature profile t'_{a2} at the outlet from Z-shaped channels. Under actual conditions, even in the absence of finning in parts with crossflow, only partial mixing of air occurs in these zones. It could be expected that, with respect to efficiency, such systems occupy an intermediate position between the systems without and with complete air mixing considered in the present article.

In conclusion, it should be mentioned that the efficiency data for Z-shaped systems given in Tables 1 and 2 can also be used for calculating heat exchangers where Z-shaped channels are provided in both the air and the gas cavities and the flow deflection angle is different from 90°.

NOTATION

t_{a1}, t_{g1}, t_{a2}, and t_{g2}	are the mean-mass coolant temperatures at the inlet and outlet sections of the heat exchanger (°K);
t', t'', and t'''	are the local temperature values (°K);
$T = (t-t_{a1})/(t_{g1}-t_{a1})$	are the dimensionless temperatures;
c_{pg} and c_{pa}	are the specific heat values at constant pressure (J/kg·°K);
G_g and G_a	are the coolant discharges (kg/sec);
$\omega = c_{pg}G_g/c_{pa}G_a$	is the ratio of the water equivalents of the coolants;
k' and k''	are the heat transfer coefficients (J/m²·sec·°K);
F' and F''	are the heat-exchange surface areas (m²);
$M'_g = k'F'/c_{pg}G_g$ and $M''_g = k''F''/c_{pg}G_g$	are the parameters of the Z-shaped heat exchanger;
M and $\bar{M}$	are the parameters of the Z-shaped heat exchanger determined by relationships (9) and (10);

η_r is the degree of regeneration defined by relationship (8);

η_r'' is the efficiency of the counterflow part defined by relationship (7).

Subscripts and Superscripts

a is the air;

g is the gas;

1 and 2 are the inlet and outlet sections, respectively;

', ", and ''' are parts of the Z-shaped heat exchanger.

LITERATURE CITED

1. M. A. Mikheev, Fundamentals of Heat Transfer [in Russian], GÉI, Moscow–Leningrad (1949).
2. W. M. Kays and A. L. London, Compact Heat Exchangers, [Russian translation], McGraw (1964).
3. D. M. Smith, Engineering, No. 3590, 479-481 (1934).